Exercise 10F: Solutions

1 a i 2π

ii 4π

iii -4π

b i $2\pi \pm \frac{2\pi}{3} = \frac{4\pi}{3}, \frac{8\pi}{3}$

ii $4\pi \pm \frac{2\pi}{3} = \frac{14\pi}{3}, \frac{10\pi}{3}$

iii $-4\pi \pm \frac{2\pi}{3} = -\frac{14\pi}{3}, -\frac{10\pi}{3}$

2 a $2n\pi \pm \frac{\pi}{6}, n \in \mathbb{Z}$

b $\frac{2n\pi}{3} + \frac{\pi}{9}$ or $\frac{2n\pi}{3} + \frac{2\pi}{9}, n \in \mathbb{Z}$

c $n\pi + \frac{\pi}{3}, n \in \mathbb{Z}$

3 a
$$\sin x = 0.5$$
$$x = 2n\pi + \sin^{-1}(0.5) \text{ or } (2n+1)\pi - \sin^{-1}(0.5)$$
$$= 2n\pi + \frac{\pi}{6} \text{ or } (2n+1)\pi - \frac{\pi}{6}$$
$$= \frac{12n\pi}{6} + \frac{\pi}{6} \text{ or } \frac{6(2n+1)\pi}{6} - \frac{\pi}{6}$$
$$= \frac{(12n+1)\pi}{6} \text{ or } \frac{(12n+5)\pi}{6}$$
$$x = \frac{(12n+1)\pi}{6} \text{ or } \frac{(12n+5)\pi}{6}$$

When $n = 0$, $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$.

b
$$\cos 2x = \frac{\sqrt{3}}{2}$$
$$2x = 2n\pi \pm \frac{\pi}{6}$$
$$x = \frac{2n\pi}{2} \pm \frac{\pi}{12}$$
$$= \frac{12n\pi \pm \pi}{12}$$
$$= \frac{(12n \pm 1)\pi}{12}$$
$$x = \frac{(12n \pm 1)\pi}{12}$$

When $n = 0$, $x = \pm\frac{\pi}{12}$

When $n = 1$, $x = \frac{11\pi}{12}$ or $x = \frac{13\pi}{12}$

Hence $x = \frac{\pi}{12}, \frac{11\pi}{12}$

c

$$\tan 2x = -\frac{3}{\sqrt{3}} = -\sqrt{3} \quad x = \frac{(3n-1)\pi}{6}$$

$$x = n\pi - \frac{\pi}{3}$$

$$= \frac{3n\pi - \pi}{6}$$

$$= \frac{(3n-1)\pi}{6}$$

When $n = 1$, $x = \frac{\pi}{3}$

When $n = 2$, $x = \frac{5\pi}{6}$

Hence $x = \frac{\pi}{3}, \frac{5\pi}{6}$.

4 $\frac{-11\pi}{6}, \frac{-7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

5 $\frac{-\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$

6 **a** $x = n\pi - \frac{\pi}{6}$ or $x = n\pi - \frac{\pi}{2}, n \in \mathbb{Z}$

b $x = \frac{n\pi}{2} - \frac{\pi}{12}, n \in \mathbb{Z}$

c $x = 2n\pi + \frac{5\pi}{6}$ or $x = 2n\pi - \frac{\pi}{2}, n \in \mathbb{Z}$

7 $\cos\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$$2x + \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$$2x = 2n\pi - \frac{\pi}{4} \pm \frac{\pi}{4}$$

$$= 2n\pi \text{ or } 2n\pi - \frac{\pi}{2}$$

$$x = n\pi \text{ or } x = \frac{(4n-1)\pi}{4}$$

$$= -\frac{5\pi}{4}, -\pi, -\frac{\pi}{4}, 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}$$

8 $\tan\left(\frac{\pi}{6} - 3x\right) = \frac{1}{\sqrt{3}}$

$$\frac{\pi}{6} - 3x = n\pi + \frac{\pi}{6}$$

$$-2x = n\pi$$

$$x = -\frac{n\pi}{3}$$

This is equivalent to $x = \frac{n\pi}{3}$ where $n \in Z$.

$x = -\pi, -\frac{2\pi}{3}, -\frac{\pi}{3}, 0$

9 $\sin 4\pi x = \dfrac{\sqrt{3}}{2}$

$$
\begin{aligned}
4\pi x &= 2n\pi - \frac{\pi}{3} \\
&= \frac{6n\pi - \pi}{3} \\
&= \frac{(6n-1)\pi}{3} \\
\text{or } 4\pi x &= (2n+1)\pi - \frac{\pi}{3} \\
&= \frac{6n\pi + 3\pi + \pi}{4} \\
&= \frac{(6n+4)\pi}{3} \\
x &= \frac{6n-1}{12} \text{ or } \frac{3n+2}{6} \\
x &= -\frac{2}{3}, -\frac{7}{12}, -\frac{1}{6}, -\frac{1}{12}, \frac{1}{3}, \\
&\quad \frac{5}{12}, \frac{5}{6}, \frac{11}{12}
\end{aligned}
$$